

# C.U.SHAH UNIVERSITY

## Winter Examination-2015

**Subject Name :** Advance Mathematics

**Subject Code :** 2TE02AMT2

**Branch :** Diploma(All)

**Semester :** 02

**Date :** 19/11/2015

**Time :** 10:30 am To 1:30 pm

**Marks :** 70

**Instructions:**

- (1) Use of Programmable calculator & any other electronic instrument is prohibited.
- (2) Instructions written on main answer book are strictly to be obeyed.
- (3) Draw neat diagrams and figures (if necessary) at right places.
- (4) Assume suitable data if needed.

**Q-1**

**Attempt the following questions:**

**(14)**

- a) Magnitude of  $3i - 4j - 5k$  is \_\_\_\_\_.  
 (a)  $2\sqrt{5}$  (b) 0 (c) -6 (d)  $5\sqrt{2}$
- b) If  $\mathbf{a} = \mathbf{i} + \mathbf{j}$  and  $\mathbf{b} = \mathbf{j} - \mathbf{k}$  then  $\mathbf{a} \cdot \mathbf{b} =$  \_\_\_\_\_.  
 (a) 0 (b) -1 (c) 1 (d) none of these
- c) If  $\mathbf{a} = \mathbf{i} - \mathbf{j} + \mathbf{k}$  then  $\hat{\mathbf{a}} =$  \_\_\_\_\_.  
 (a)  $\frac{1}{\sqrt{2}}(1, -1, 1)$  (b)  $\frac{1}{\sqrt{3}}(1, -1, 1)$  (c)  $\sqrt{3}$  (d) none of these
- d)  $\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^{4n} =$  \_\_\_\_\_.  
 (a)  $e^4$  (b)  $e$  (c) 1 (d) none of these
- e)  $\lim_{x \rightarrow 0} \frac{\sin \sqrt{x}}{\sqrt{x}} =$  \_\_\_\_\_.  
 (a) 0 (b)  $e$  (c)  $\frac{1}{2}$  (d) 1
- f)  $\lim_{x \rightarrow 0} \frac{5^x - 1}{x} =$  \_\_\_\_\_.  
 (a) 0 (b)  $e^5$  (c)  $\log_e 5$  (d) 1
- g) If  $f(x) = x^2 - 1$  then  $f(-1) =$  \_\_\_\_\_.  
 (a) -2 (b) 0 (c) -1 (d) none of these
- h)  $\frac{d(6^x)}{dx} =$  \_\_\_\_\_.  
 (a)  $6^x \log_e 6$  (b)  $\log_e 6$  (c)  $x^6 \log_e 6$  (d)  $6^x \log_e x$



- i)  $\frac{d(\sqrt{x})}{dx} = \underline{\hspace{2cm}}$   
 (a)  $2x^{\frac{1}{2}}$  (b)  $\frac{1}{2}\sqrt{x}$  (c)  $\frac{2}{\sqrt{x}}$  (d)  $\frac{1}{2\sqrt{x}}$
- j)  $\frac{d(x^e)}{dx} = \underline{\hspace{2cm}}$   
 (a)  $x^{e-1}$  (b)  $xe^{x-1}$  (c)  $ex^{e-1}$  (d)  $e^{x-1}$
- k)  $\int e^x dx = \underline{\hspace{2cm}}$   
 (a)  $\log x$  (b)  $e^x$  (c)  $\frac{e^x}{\log x}$  (d) none of these
- l)  $\int \frac{1}{x^2+1} dx = \underline{\hspace{2cm}}$   
 (a)  $\tan^{-1} x + c$  (b)  $\cos^{-1} x + c$  (c)  $\cot^{-1} x + c$  (d)  $\sin^{-1} x + c$
- m)  $\int \frac{1}{\sqrt{1-x^2}} dx = \underline{\hspace{2cm}}$   
 (a)  $\tan^{-1} x + c$  (b)  $\cos^{-1} x + c$  (c)  $\cot^{-1} x + c$  (d)  $\sin^{-1} x + c$
- n)  $\int_0^1 x^2 dx = \underline{\hspace{2cm}}$   
 (a) 2 (b) 1 (c)  $\frac{1}{3}$  (d) none of these

**Attempt any four questions from Q-2 to Q-8**

- Q-2 Attempt all questions (14)**
- a) If  $a = i + 2j - k$ ,  $b = 3i + j + 2k$  and  $c = -2i - j + 5k$  then find  $|2a + 3b - c|$ . (5)
- b) If  $a = 2i - j$ ,  $b = i + 3j - 2k$  then obtain  $|(a + b) \times (a - b)|$ . (5)
- c) For what value of  $p$ , vectors  $2i + 3j - k$  and  $pi - j + 3k$  are perpendicular to each other? (4)
- Q-3 Attempt all questions (14)**
- a) A particle moves from the point  $3i - 2j + k$  to the point  $i + 3j - 4k$  under the effect of constant forces  $i - j + k$ ,  $i + j - 3k$  and  $4i + 5j - 6k$ . Find the work done. (5)
- b) Prove that angle between two vectors  $i + 2j$  and  $i + j + 3k$  is  $\sin^{-1}\left(\sqrt{\frac{46}{55}}\right)$ . (5)
- c) Find unit vector which is perpendicular to  $a = 5i + 7j - 2k$  and  $b = 3i + j - 2k$ . (4)
- Q-4 Attempt all questions (14)**
- a) Prove that  $\lim_{x \rightarrow 3} \frac{\sqrt{x+2} - \sqrt{5}}{\sqrt{x+4} - \sqrt{7}} = \frac{\sqrt{35}}{5}$  (5)
- b) Evaluate:  $\lim_{\theta \rightarrow 0} \frac{\operatorname{cosec}\theta - \cot\theta}{\theta}$  (5)



- c) Prove that If  $f(x) = \log\left(\frac{x-1}{x}\right)$  then prove that  $f(x) + f(-x) = f(x^2)$ . (4)

**Q-5 Attempt all questions (14)**

- a) Find derivative of  $f(x) = x^3$  using definition. (5)
- b) Find  $\frac{dy}{dx}$  if  $y = \log\left[x + \sqrt{x^2 + a^2}\right]$ . (5)
- c) Find  $\frac{dy}{dx}$  if  $y = \frac{a + b\sin x}{a\sin x + b}$ . (4)

**Q-6 Attempt all questions (14)**

- a) Find  $\frac{dy}{dx}$  if  $e^x + e^y = e^{x+y}$ . (5)
- b) The equation of motion of a particle is  $S = 2t^3 + 3t^2 - 12t + 5$ . Find velocity at  $t = 0$  and acceleration at  $t = 1$ . (5)
- c) Find  $\frac{dy}{dx}$  if  $y = (\sin x)^{\tan x}$ . (4)

**Q-7 Attempt all questions (14)**

- a) Evaluate:  $\int \frac{(1-3x)^2}{x^3} dx$  (5)
- b) Evaluate:  $\int x \log x dx$  (5)
- c) Evaluate:  $\int \sin^4 x \cos x dx$  (4)

**Q-8 Attempt all questions (14)**

- a) Prove that  $\int_0^{\frac{\pi}{2}} \frac{\sec x}{\sec x + \operatorname{cosec} x} dx = \frac{\pi}{4}$  (7)
- b) Find the area of standard ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  (7)

